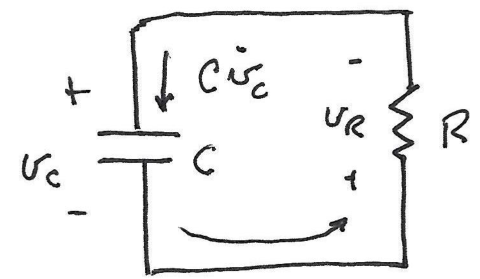


$$v_R = RC\dot{v}_c$$

$$v_c + v_R = 0$$

$$v_c + RC\dot{v}_c = 0$$

$$\dot{v}_c + \frac{1}{RC} v_c = 0$$

Assume $\quad v_c = Ke^{\alpha t}$

$$\alpha K e^{\alpha t} + \frac{1}{RC} K e^{\alpha t} = 0$$

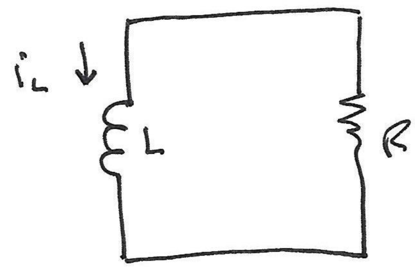
$$K e^{\alpha t} \left(\alpha + \frac{1}{RC}\right) = 0$$

$$\alpha + \frac{1}{RC} = 0 \qquad \text{characteristic equation}$$

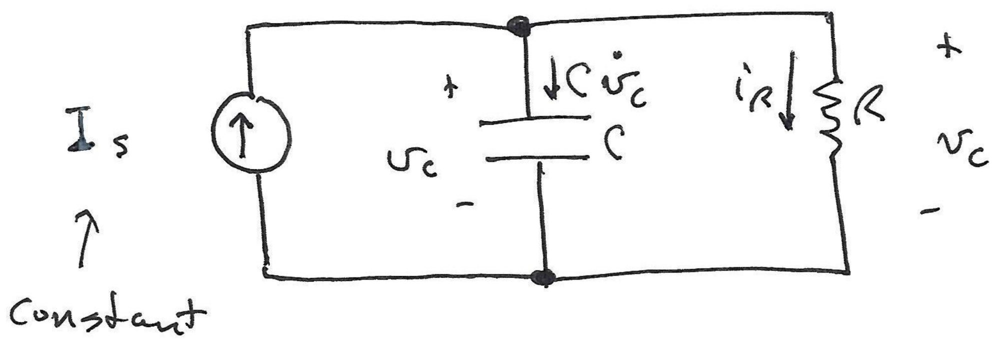
$$\alpha = -\frac{1}{RC}$$

$\tau = RC$ is called the time constant

Similarly, for an RL circuit:



$i_L(t) = K e^{-\alpha t}$ where $\alpha = \frac{R}{L}$ or $\tau = \frac{L}{R}$

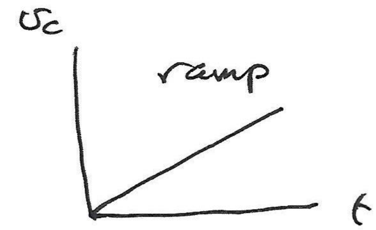


$i_R = \frac{v_C}{R}$

$$C\dot{v}_C + i_R = I_s$$

$$C\dot{v}_C + \frac{v_C}{R} = I_s$$

$$\dot{v}_c + \frac{1}{RC} v_c = \frac{I_s}{C}$$



$$v_c = At$$

$$v_c = K_1 e^{\alpha t} + K_2$$

$$\underbrace{\alpha K_1 e^{\alpha t} + 0}_{\dot{v}_c} + \underbrace{\frac{K_1}{RC} e^{\alpha t} + \frac{K_2}{RC}}_{\frac{1}{RC} v_c} = \frac{I_s}{C}$$

$$K_1 e^{\alpha t} \left(\alpha + \frac{1}{RC} \right) + \frac{K_2}{RC} = \frac{I_s}{C}$$

$$K_1 e^{\alpha t} \left(\alpha + \frac{1}{RC} \right) = 0$$

$$\alpha + \frac{1}{RC} = 0$$

characteristic equation

Same as before!

$$\frac{K_2}{RC} = \frac{I_s}{C} \Rightarrow K_2 = R I_s$$

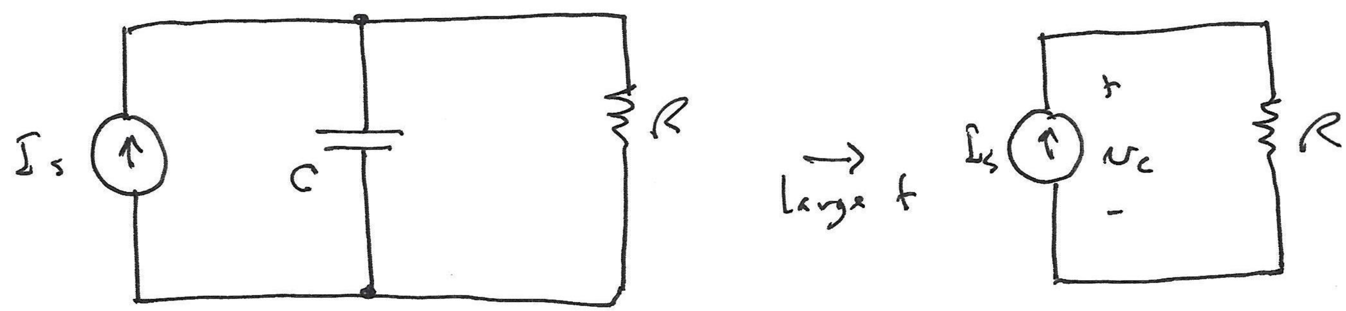
$$v_c(t) = K_1 e^{\alpha t} + R I_s$$

$$\lim_{t \to \infty} v_c(t) = R I_s = \underbrace{v_c(\infty)}_{\text{final value of } v_c(t)}$$

$$v_c(t) = K_1 e^{\alpha t} + v_c(\infty)$$

For large values of t, $i_c(t) \to 0$

The capacitor behaves like an open circuit for large value of t.



$$v_c(\infty) = R I_s$$

How do we find K_1?

$$\alpha = -\frac{1}{RC} \quad \text{or} \quad \tau = \frac{1}{RC}$$

$$v_c(\infty) = RI_s$$

$$v_c(t) = K_1 e^{\alpha t} + v_c(\infty)$$

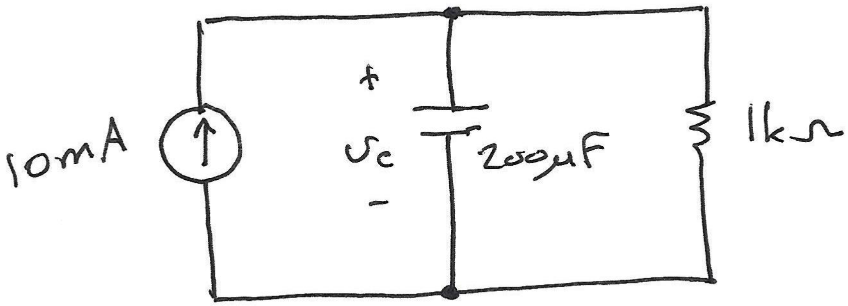
$$\Rightarrow \quad v_c(t) = K_1 e^{-t/RC} + RI_s$$

$$v_c(0) = K_1 + v_c(\infty)$$

$$\therefore \quad K_1 = v_c(0) - v_c(\infty)$$

So, in general, for an RC circuit,

$$\boxed{v_c(t) = [v_c(0) - v_c(\infty)]\, e^{-t/RC} + v_c(\infty)}$$



$v_c(0) = 0$

Determine $v_c(t)$ for $t \geq 0$.

We know:

$$v_c(t) = [v_c(0) - v_c(\infty)]\, e^{-t/RC} + v_c(\infty)$$

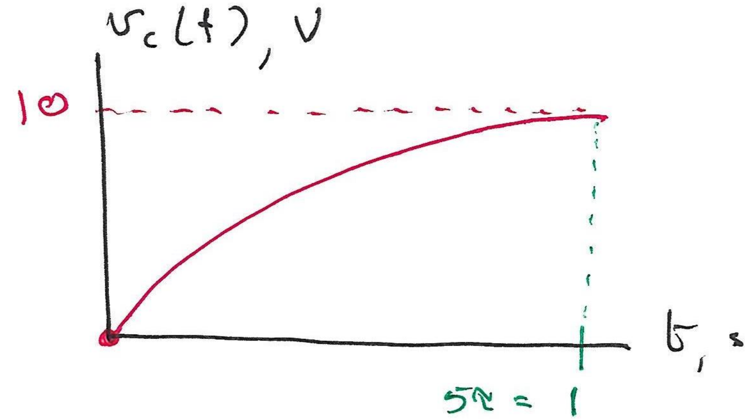
Here $\tau = RC = 1000 \times 200 \times 10^{-6} = 0.2\ s$

$v_c(0) = 0$ (given)

$v_c(\infty) = (1k\Omega)(10mA) = 10V$

$$\therefore \quad v_c(t) = (0 - 10)\, e^{-t/0.2} + 10$$

$$= -10e^{-5t} + 10 \quad V, \ t \geq 0$$



Revise the problem to have $v_c(0) = 5$ V

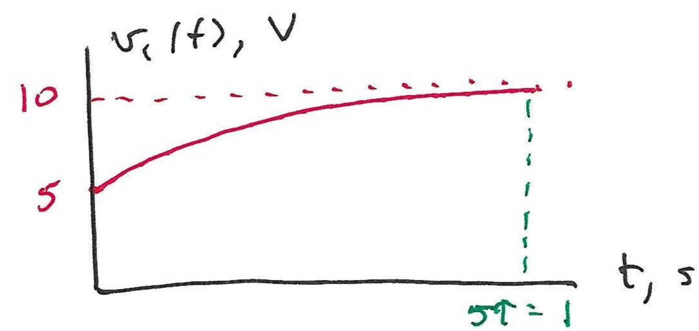
$$v_c(t) = [v_c(0) - v_c(\infty)]e^{-t/\tau} + v_c(\infty)$$

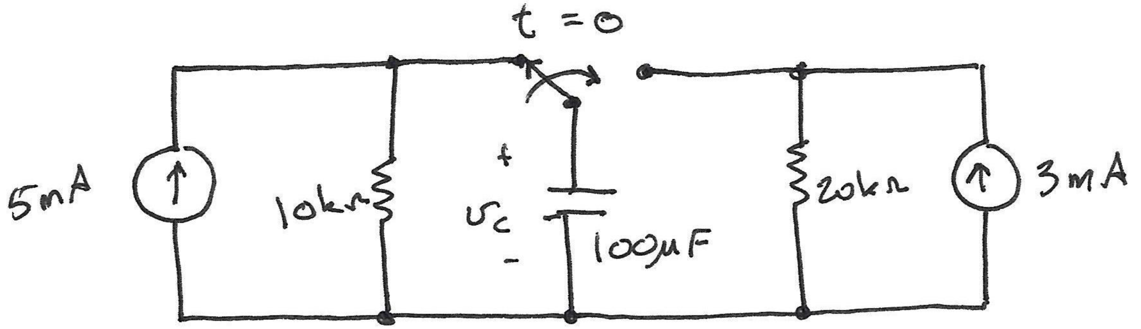
$$v_c(\infty) = 10 \text{ V}$$

$$\tau = RC = 0.2 \text{ s}$$

$$v_c(t) = [5 - 10]e^{-t/0.2} + 10$$

$$= -5e^{-5t} + 10 \quad \text{V}, \quad t \geq 0$$





Determine $v_c(t)$ for $t \geq 0$.

$\uparrow$ lhs = 1 s

1. Find $v_c(0^-)$. $v_c(0^-) = (10k\Omega)(5mA) = 50V$

2. $v_c(0^+) = v_c(0^-) = 50V$

   $v_c(0^+)$ is the initial value for the capacitor voltage after the switch is moved.

3. For the circuit on the r.h.s.:

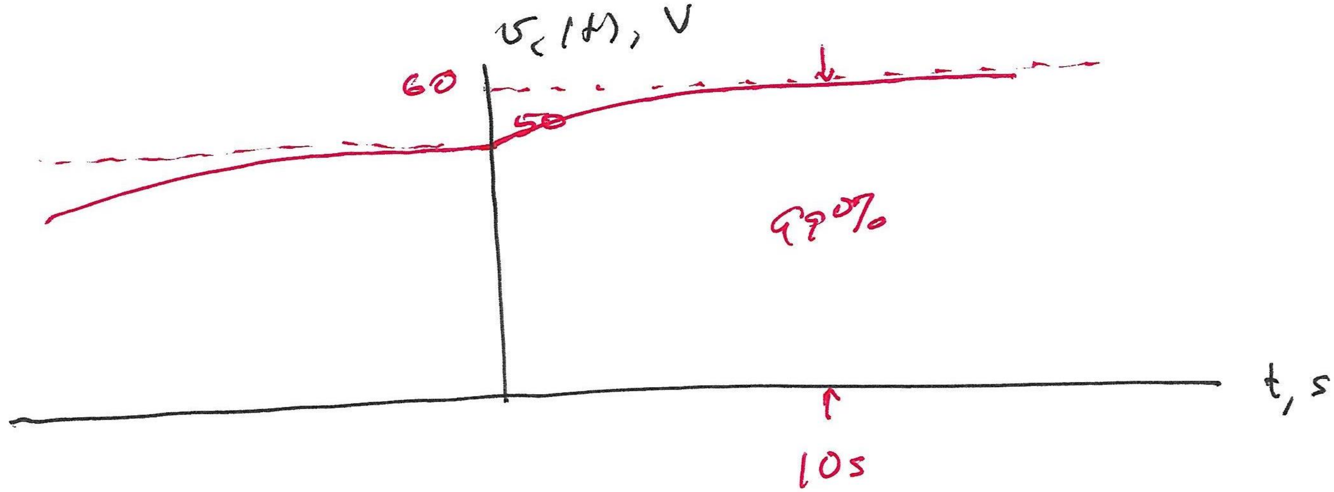
$$\tau = (20k\Omega)(100\mu F) = 2\,s$$

$$v_c(\infty) = (3mA)(20k\Omega) = 60\,V$$

$$\therefore \quad v_c(t) = [v_c(0) - v_c(\infty)]e^{-t/\tau} + v_c(\infty)$$

$$= (50 - 60)e^{-t/2} + 60$$

$$= -10e^{-t/2} + 60 \quad V, \ t \geq 0$$


$v_c(t)$, V
60
50
99%
t, s
10s